**APPENDIX D**

**Method of Teaching: Lecture Method**

**Lesson:** 1

**Subject:** Mathematics

**Class:** SS2

**Sex:** Mixed

**Average age:** 16

**Group:** Control

**Duration:** 5 periods per week

**Topic:** Factoring quadratic expressions

**Instructional Materials:** Textbook and Chalkboard.

**Reference Book:** New General Mathematics for Senior Secondary School two (2).

**Teaching Method:** Lecture Method

**Behavioral Objectives:** By the end of the lesson students should be able to:

1. Identify the coefficients of terms.
2. Factorize two-termed quadratic expressions
3. Factorize three-termed quadratic expressions

**Entry Behavior:** Students are already taught how to:

1. Simplify integers and algebraic expressions
2. Multiply two linear expressions

**Introduction:** The teacher instructs his students to answer the following questions:

1. Simplify the following **a.** (–1) – (+ 4) **b.**  **c.** 2x2 – ( + x2) **d.** (2x–7) + (x2 + 3x – 5)
2. Evaluate **a.** (2x + 1) (x + 3) **b. –**2(3x2 – x + 2)

**Presentation:** The teacher presents his lesson using the following steps.

**Step 1:** The lesson begins with the identification of the coefficient of terms. The teacher explains that we can identify a particular term in an expression by using the letter, or combination of letters, involved, for example, 2x2 is ‘a term in x2’ and 3xy is ‘a term in xy’. The number in front of the letter(s) is called the coefficient. If no number is written in front of a term, the coefficient is either 1 or – 1, depending on the sign of the term.

**Activity 2:** Finding the coefficient of terms in the expansion of two linear expressions

**Example 1:** Find the coefficient of x in the expansion of (4x + 3) (x – 2)

**Solution:** (4x + 3) (x – 2) = 4x(x – 2) + 3(x – 2)

= 4x2 – 8x + 3x – 6 = 4x2 – 5x – 6

From the above result, the coefficient of x is – 5.

**Example 2:** If 2x2 + kx –14 = (x + 2) (2x – 7), find the value of k.

**Solution:** (x + 2) (2x – 7) = x(2x – 7) + 2(2x – 7)

= 2x2 – 7x + 4x – 14

= 2x2 – 3x – 14

By comparing this result with 2x2 + kx –14, k = – 3.

**Step 2:** The students will be taught the meaning of factorization and then shown how to factorize any type of quadratic expression. To factorize a quadratic expression means to write it as a product of two linear expressions called factors. The method of factorizing quadratic expressions depends on the nature of the expressions. See the below activities.

**Activity 3:** Factoring two-termed quadratic expressions

**Example 3:** Factorise the quadratic expressions **i.** x2 – 3x. **ii.** 2x2 – 6x.

1. x2 is a common factor of the terms x2 and 3x. Hence, x2 – 3x can be written as x.x – 3. x. Isolating common factor, we have x2 – 3x = x(x – 3).
2. The common factor of the terms 2x2 and– 4x is 2x. Therefore, 2x2 – 6x can be written as 2x.x – 2x.3. Isolating common factor we obtain, 2x2 – 6x = 2x(x – 3).

**Step 3:** The lesson continues with the factorization of quadratic expressions. In this step, the students will learn how to factorize three-termed expressions using the guidelines below.

**Activity 4:** Factoring three-termed quadratic expressions

**Example 4:** Factorise the quadratic expressionx2 + 5x + 6.

**Solution:** To factorize the given expression, the teacher used the following steps:

1. Multiply the coefficient of x2 and the constant term and write down the pairs of factors of the product. The product is 1× 6 = 6. The pairs of factors of 6 are (6, 1); (–6, –1); (2, 3); (–2, –3).
2. Select the pair of factors whose sum is equal to the coefficient of x and the product is 6.
3. The required pair is (2, 3) since the sum is 5 (the coefficient of x) and the product is 6.
4. Rewrite the expression by replacing 5x with 2x + 3x i.e, x2 + 5x + 6 = x2 + 2x + 3x + 6.
5. Group them and take out the common factors; (x2 + 2x) + (3x + 6) = x(x + 2) +3(x + 2).
6. Isolating common factor we obtain; (x + 2) (x + 3). Thus, x2 + 5x + 6 is factored into (x + 2) (x + 3).

**Summary:** The teacher runs through the topic and stresses some important points.

**Evaluation:** The teacher evaluates the students with the following exercises:

1. Write down the coefficients of x2 and x in the expansion of

**a.** (1 – x) (2 – 3x) **b.** (2x – 4) (3x – 5) **c.** (2x – 3) (7x – 5)

1. Factorise the following:

**a.** 24r – 3r2 **b.** 2x2 + 2 **c.** 5p2 – p **d.** x2 – 4x – 5 **e.** x2 – 10x + 9 **f.** 9 + 6x + x2

**Conclusion:** The teacher concludes his lesson by marking the students’ exercises and works out corrections

**Lesson:** 2

**Subject:** Mathematics

**Class:** SS2

**Sex:** Mixed

**Average age:** 16

**Group:** Control

**Duration:** 5 periods per week

**Topic:** Algebraic Processes 2

**Sub Topic:** Factorizing quadratic expressions where c < 0

**Instructional Materials:** Textbook and Chalkboard.

**Reference Book:** New General Mathematics for Senior Secondary School two (2).

**Teaching Method:** Lecture Method

**Behavioral Objectives:** By the end of the lesson students should be able to:

1. Factorize more difficult quadratic expressions.
2. Find the missing factor of a given quadratic expression.
3. Factorize algebraic expressions by grouping terms.

**Entry Behavior:** Students are already taught how to:

1. Identify the coefficients of terms.
2. Factorize two-termed quadratic expressions
3. Factorize three-termed quadratic expressions

**Introduction:** The teacher instructs his students to answer the following questions:

1. Write out the coefficients of each term in the expressions

**a.** 2p2 + 3pq + q2 **b.** x3 + 5x2y – y3

1. Factorize: **a.** 2ab – a2 **b.** x2 + x – 12

**Presentation:** The teacher presents his lesson using the following steps.

**Step 1:** The students will be taught how to factorize harder quadratic expressions. Examples of such are 9x2 – 54x + 81; 35 + 18 – 5x2 where the coefficient of x2 is each case, not unity, and 2x2 + 7x2y – 15y2 which contains more than one letter.

**Activity 1:** Factorizing harder quadratic expressions.

**Example 1:** Factorize 8x2 – 14x – 9.

1. Step 1: The product of 8x2 × (– 9) = – 72x2. The pairs of factors are: (– 72, +1); (– 36, +2); (– 18, +4)
2. Step 2: The sums of factors are: – 71, – 34 and – 14.
3. Step 3: Hence, the required pair is (– 18, +4)
4. Step 4: Group and factorize

8x2 – 14x – 9 = 8x2 + 4x – 18x – 9

= (8x2 + 4x) – (18x + 9)

= 4x(2x + 1) – 9(2x + 1)

= 4(2x + 1) (4x – 9)

**Step 2:** After the students have learned how to factorize quadratic expressions, the teacher will use the concepts of grouping, factorizing, and isolating the common factor which they have learned in the previous steps to teach them how to factorize expressions containing four terms.

**Activity 2:** Factorizing algebraic expressions by grouping terms

**Example 2:** Factorize 3x – 2dy + 3y – 2dx

**Solution:**

1. Step 1: The terms 3x and 3y both have 3 in common. The terms 2dx and 2dy both have 2d in common. Rearrange the expression in this order.

3x – 2dy + 3y – 2dx = 3x + 3y – 2dx – 2dy

1. Step 2: Group the terms and factorize;

(3x + 3y) – (2dx + 2dy) = 3(x + y) – 2d(x + y)

1. Step 3: Isolating the common factor, we obtain (x + y) (3 – 2d).

Thus, 3x – 2dy + 3y – 2dx = (x + y) (3 – 2d).

**Step 3:** At this stage, the students will be taught how to look for a missing factor in a given quadratic expression.

**Activity 3:** Finding a missing factor of a given quadratic expression

**Example 3:** If (x − 3) is one of the factors of x2 + 4x – 21, what is the other factor?

**Solution:**

1. Step 1:Theexpression can written as x2 + 4x – 21= (x − 3) (x + ).
2. Step 2: Using the idea that −3 and + are numbers when multiplied yields −21(i.e factors of −21) and when added gives 4, we have −3 × = −21 (Answer = 7). That is, −3 × 7 = −21 (product) and −3 + 7 = 4 (sum).
3. Step 3: Thus, x2 + 4x – 21= (x − 3) (x +7)

**Summary:** The teacher wraps up the lesson and stresses some important points.

**Evaluation:** The teacher evaluates the students with following exercises:

1. Factorize the following **a.** 4x2 − 12x + 9 **b.** 12x2 − 15x + 9 **c.** 15x2 − 33x – 36
2. If (x − 4) is one of the factors of 3x2 – 11x – 4, what is the other factor?
3. What are the factors of **a.** ax – ay + 6y– 6x **b.** ab **+** ac **– (**b + c**)**2?

**Conclusion:** The teacher concludes his lesson by marking the students’ exercises and working out corrections.

**Lesson:** 3

**Subject:** Mathematics

**Class:** SS2

**Sex:** Mixed

**Average age:** 16

**Group:** Control

**Duration:** 5 periods per week

**Topic:** Factoring perfect squares, difference of two squares, and simplifying algebraic fractions using factorization method.

**Instructional Materials:** Textbook and Chalkboard.

**Reference Book:** New General Mathematics for Senior Secondary School two (2).

**Teaching Method:** Lecture Method

**Behavioral Objectives:** By the end of the lesson students should be able to:

1. Factorize perfect squares quadratic expressions.
2. Make quadratic expressions perfect squares.
3. Factorize difference of two squares.
4. Simplify algebraic fractions by reducing them to their lowest terms using the factorization method.

**Entry Behavior:** Students are already taught how to:

1. Factorize more difficult quadratic expressions.
2. Find the missing factor of a given quadratic expression.
3. Factorize algebraic expressions by grouping terms.

**Introduction:** The teacher instructs his students to answer the following question:

Factorize **a.** px + pq – 6x – 6q **b.** ab +xy – ay – bx

**Presentation:** The teacher presents his lesson using the following steps.

**Step 1:** The lesson begins with an explanation of perfect square quadratic expression as follows: A quadratic expression is a perfect square if it can be expressed as a product of two linear factors that are the same. Examples are 4x2 + 4x + 1 = (2x + 1)2; x2 + 18x + 81 = (x + 9)2 and so on. Thus, a perfect square quadratic expression is a special expression that can be factored into two identical factors as given in the examples.

**Activity 2:** Factoring perfect square quadratic expressions

**Example 1:** Factorise the quadratic expression 4x2 – 4x + 1

**Solution:**

1. Step 1: Multiply the coefficient of x2 and the constant term and write down the pairs of factors of the product. The product is 4, a perfect square.
2. Step 2: Determine the square root of the product whose sum is equal to the coefficient of x.

= + 2 or – 2, i.e. 4 = (+ 2)(+ 2) or (– 2)(– 2)

Sum of factors: (+ 2) + (+ 2) = + 4

(– 2) + (– 2) = – 4.

Hence, 4x = – 2x – 2x

1. Step 3: Write down the quadratic equation as follows

4x2 – 2x – 2x + 1

1. Step 5: Group the terms and factorize

(4x2 – 2x) – (2x – 1) = 2x (2x – 1) – 1(2x – 1)

Isolating common factor, we obtain

(2x – 1) (2x – 1) = (2x – 1)2

**Step 2:** Students will also learn how to make a quadratic expression a perfect square as shown in the example below.

**Activity 2:** Making a quadratic expression a perfect square

**Example 2:** What must be added to x2 + 6x to make the expression a perfect square?

**Solution:** Suppose x2 + 6x + k is a perfect square and that it is equal to (x + a)2.

i.e., let x2 + 6x + k = (x + a)2

Then, x2 + 6x + k = x2 + 2ax + a2

By comparing coefficients of x

2a = 6 ⇒ a = 3

By comparing constant terms: K = a2 ⇒ k = 32 = 9

Therefore 9 must be added to the expression. Thus, x2 + 6x + 9 = (x + 3)2

From the example above, it should be noted that the quantity to be added is the square of half of the coefficient of x. That is, the square of half of + 6 = (.)2 = + = + 9

**Step 3:** In this step, the students will be made to know that quadratic expressions can occur as a difference of two squares, as in x2 – 4. The terms x2 and 4 are perfect squares; hence, the expression x2 – 4 is called a perfect square. The teacher will teach them how to use the fundamental identity A2 – B2 = (A – B) (A + B), to factorize difference of two squares.

**Activity 3:** Factoring difference of two squares.

**Example 3:** Factorizex2 – 16.

**Solution:**

1. Step 1: Write x2 – 16 as a difference of two squares. i.e. x2 – 16 = x2 – 42
2. Step 2: x2 – 42 = x.x – 4.4. Thus, the factors are (x – 4) (x + 4)

**Example 4:** Factorize9 – 36x2.

**Solution:**

1. Step 1: There is an obvious factor 9. By isolating this factor, we obtain 9 (1 – 4x2).
2. Step 2: The terms in the bracket are the difference between the squares of 1 and 2x.

i.e., 9 (1 – 4x2) = 9 [(1)2 – (2x)2]

Thus, the factors are 9 (1 – 2x) (1 + 2x).

**Step 4:** Having learned different types of quadratic expressions as well as various factorization methods, the students will then learn how to apply the knowledge to the simplification of algebraic fractions.

**Activity 4:** Simplifying algebraic fractions.

**Example 5:** Simplify

**Solution:** By factoring in the top and bottom expressions we have,

Divide through by the common factor result into

Thus, reduces to .

**Summary:** The teacher runs through the topic and stresses some important points:

1. If x2 + bx + c is a perfect square, then c = ( × b) 2
2. When factorizing, first look for an obvious factor
3. To simplify an algebraic fraction, always reduce its top and bottom expressions by factorizing.

**Evaluation:** The teacher evaluates the students with the following exercises:

1. Factorize the following quadratic expressions:

**a.** 16x2− 49 **b.** 25q2− 1 **c.** a2− 100 **d.** x2 − 2x + 1 **e.** 4y2 − 12y + 9

1. Simplify completely by dividing out the common factor in each case. The first one has been done for you.

**a.**==**b.c.d.** .

**Conclusion:** The teacher concludes his lesson by marking the students’ exercises and working out corrections.

**Lesson:** 4

**Subject:** Mathematics

**Class:** SS2

**Sex:** Mixed

**Average age:** 16

**Group:** Control

**Duration:** 5 periods per week

**Topic:** Quadratic Equations

**Instructional Materials:** Textbook and Chalkboard.

**Reference Book:** New General Mathematics for Senior Secondary School two (2).

**Teaching Method:** Lecture Method

**Behavioral Objectives:** By the end of the lesson students should be able to:

1. Solving quadratic equations by factorization
2. Solving quadratic equations with irrational roots
3. Solving quadratic equations by completing the square
4. Solving quadratic equations by using the quadratic formula

**Entry Behavior:** Students are already taught how to:

1. Factorize more difficult quadratic expressions.
2. Find the missing factor of a given quadratic expression.
3. Factorize algebraic expressions by grouping terms.

**Introduction:** The teacher instructs his students to answer the following questions:

1. What must be added to x2 + 3x to make it a perfect square? Factorize the result.
2. Determine whether the following expressions are perfect squares or not

**a.** x2 − 18x + 81 **b.** 4y2 − 12y + 9 **c.** 9a2 + 24a + 16

1. Simplify
2. Using the fundamental identity a2 − b2 = (a + b) (a − b), factorize7x2 − 63

**Presentation:** The teacher presents his lesson using the following steps.

**Step 1:** The lesson begins with the introduction to the solutions of quadratic equations. A quadratic equation is an algebraic expression in the form ax2 + bx + c = 0, where a, b and c are constants and a ≠ 0. This means that quadratic equation can take any form so far the value of ‘a’ is not zero. The solutions of any quadratic equation are the values of x (or whatever letter is involved). These are sometimes called the roots of the equation.

**Activity 1:** Solving quadratic equations by factorization

To solve a quadratic equation means to find the values of the unknowns called the roots of the equation. To find the roots by factorization:

1. Arrange the equation in the form ax2 + bx + c = 0
2. Factorize the LHS if possible (i.e to write it as a product of two linear expressions)
3. Using the principle that, if a × b = 0, then either a = 0 or b = 0 (or both a and b are 0) to find the roots.

**Example 1:** Solve the equation 2x2 = 3x + 15.

**Solution:**

1. Step 1:Rearrange the terms in such a way that the equation will be in the form

ax2 + bx + c = 0. i.e. 2x2 – 3x – 15 = 0

1. Step 2: Factorize the LHS.

2x2 – 3x – 15 = 0

2x2 + 2x – 5x – 15 = 0

2x (x + 1) – 5 (x + 1) = 0

(2x – 5) (x + 1) = 0

1. Step 3: Apply the zero product principle

i.e. if (2x – 5) (x + 1) = 0, then

Either 2x – 5 = 0 or x + 1 = 0

⇒ 2x = 5 or x = – 1

⇒ x = 5/2 or x = – 1

**∴** The roots of the equation are x = 5/2 and x = – 1

**Activity 2:** Solving quadratic equation with irrational roots.

**Example 2:** Solve the equation (x – 3)2 = 7

**Solution:**

1. Step 1: Take the square roots of both sides, x – 3 = ±
2. Step 2: Make x the subject by adding 3 to both sides.

i.e. x – 3 + 3 = 3 ±

x = 3 ±

⇒ x = 3 + or x = 3 –

The roots are irrational because they cannot be written as the ratio of two integers. The roots may be found approximately by writing ± as ± 2.65.

**Example 3:** Solve the equation x (x + 3) (x − 5)2 = 0

**Solution:** If x (x + 3) (x − 5)2 = 0, then

Either x = 0, x + 3 = 0 or x − 5 = 0 twice

⇒ x = 0, x = − 3 or x = 5

⇒ x = 0 or − 3 or 5 twice

**Step 2:** Herethe students will learn how to find the roots of quadratic equations by completing the square.

**Activity 3:** Findingthe roots of quadratic equations by completing the square.

The solutions of quadratic equations by completing the square are found using the following procedures.

1. Bring all terms in the unknown variable, say, x, to one side, and all the terms without x to the other side.
2. Divide through by the coefficient of x2.
3. Take half the coefficient of x, square it and add it to both sides.
4. Simplify the right-hand side of the equation (which does not involve x).
5. Find the square root of both sides, remembering the positive and negative values.
6. Hence, find the values of x.

**Example 4:** Solve the equation x2 + 6x = 27

1. Step 1: Add ( × 6) 2 = 32 to each side, x2 + 6x + 32 = 27 + 32
2. Step 2: Simplify the right hand side, x2 + 6x + 32 = 27 + 9

" = 36.

1. Step 3: Factorize the left hand side, (x + 3)2 = 36
2. Step 4: Find the square root of both sides, x + 3 = ± 6
3. Step 5: Find the values of x, x = – 3 ± 6 ⇒ x = – 3 + 6 or –3 + 6 or – 3 – 6

x = 3 or – 9.

**Step 3:** In addition to the preceding methods of solving quadratic equations, the students will be taught how to use a quadratic formula to solve a quadratic equation that does not factorize.

**Activity 4:** Deriving the quadratic formula

The quadratic formula can be derived by solving the general form of a quadratic equation using the method of completing the square. For the general quadratic equation ax2+ bx + c = 0, where a, b, c are any real numbers with ‘a’ at least non-zero, we have after division by a and a slight rearrangement of terms

+x =

The addition of the quantity ()2 to both sides, makes the left-hand side a perfect square, thus: +x + ()2 = + ()2

Factorizing the LHS, (x +)2 = +

Simplifying the RHS, (x +)2 = −

(x +)2 =

If x2 = c, then x = ±. Thus x + = ±√()

Then the roots of the equation are: x = ±

That is, x =

**Activity 5:** Using the quadratic formula

**Example 5:** Find correct to 2 decimal places, the roots of the equation 3x2 – 5x – 7 = 0

**Solution:**

Comparing 3x2 – 5x – 7 = 0 with ax2+ bx + c = 0; *a* = 3, b = – 5, c = – 7.

x =

x =

x =

x =

x = or x =

x = or x =

**Summary:** The teacher wraps up the lesson and stresses the following important points to ensure mastery of the objectives.

1. To find the roots of a quadratic equation by factorization use the steps in activity 1
2. To solve a quadratic equation by completing the square use the steps in activity 3
3. To solve a quadratic equation with formula, always:

1. Rewrite the equation in the standard form of ax2+ bx + c = 0

2. Write down the values of *a*, b,and cas *a* = …, b= …., c= ….

3. Write down the quadratic formula:

x =

4. Then, write brackets where the letters were:

x =

5. Write the values of *b*, *b*, *a*, *c,* and *a* into the brackets.

6. Use a calculator to work out the answer.

**Evaluation:** The teacher evaluates the students with the following exercises:

1. Solve the following equations by factorization: x (2x + 1) = 0, x2 +3x = 0, x (x + 9) + 1 (x + 9) = 0, 14 – 5a – a2 = 0 and 1 + 3x – 10x2 = 0.
2. Solve the following equations by **a.** completing the square **b.** the use of the quadratic formula: x2 + 5x = 24, 2x2 – 5x + 2 = 0 and 5x2 – 7x + 1 = 0.

**Conclusion:** The teacher concludes his lesson by marking the students’ exercises and working out corrections.